Nearly Optimal Control of a Pilot Plant Distillation Column

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A nearly optimal on-off control strategy, previously applied to a rectification column, has been successfully implemented on a full distillation column. The controller employs a linear combination of temperature measurements to adjust the reflux flow rate in order to maintain a desired overhead product composition. A simple relationship between the control coefficients is shown to be characteristic of countercurrent systems. This relationship allows the controller to be tuned on-line by the adjustment of a single parameter, thereby eliminating the need for a dynamic model of the process and for lengthy design calculations.

The performance of the tuned on-off controller is compared with that of a tuned proportional plus integral controller and a single measurement on-off controller for feed flow disturbances and set point changes. Results show that the multi-measurement on-off control system is superior for both types of disturbances.

This work extends that reported by Brosilow and Handley (*1*) who applied a nearly optimal on-off control algorithm to a pilot scale rectification column separating a binary mixture. The controller used a linear combination of the temperatures on each tray to adjust the reflux flow so as to maintain a desired overhead product purity. The results presented in (*1*) show that the on-off controller successfully maintained a constant product purity to within the accuracy of the measuring system, even in the face of vapor flow rate disturbances exceeding 200%. No comparison of the on-off control system with other control systems was presented.

In this work the on-off control algorithm is compared to proportional plus integral control and to a single measurement on-off controller. The same column and binary system (water-methylcellosolve*) used in (*1*) are used in this study. However, in the present study the column was operated as a full distillation column rather than as a rectification section. Also, modifications in the measuring and transducing systems permitted much tighter control of the overhead product composition than that previously obtainable.

An important practical limitation of the control system proposed in (*1*) is that the computation of the coefficients in the control law requires a significant amount of process data which often is not readily available. It is therefore highly desirable to develop a simple on-line tuning procedure for selecting the controller parameters with a minimum of a priori information. The results presented in (*1*) do in fact open up the possibility of constructing such a procedure since it was found empirically that the controller parameters correlated as a simple exponential function of the position of the measurement in the column.

In the next-to-last section we show that the exponential relationship between the parameters of the on-off controller and the column location which was observed in (*1*) is in fact characteristic of all columns for which the pseudo equilibrium curve can be approximated by a straight line over the top stages of the column. The following section shows how this relationship is exploited to obtain a simple single parameter tuning algorithm.

* methyl cellosolve = ethylene glycol monomethylether.

THE CONTROL SYSTEM

The on-off control law is obtained by solving the following problem.

$$\underset{L(t)}{\text{Minimize}} \quad x_c^2(t) \tag{1}$$

subject to

$$L_{\text{Min}} \leq L(t) \leq L_{\text{Max}} \tag{2}$$

$$\dot{x}(t) = A\,x(t) + B\,L(t) \tag{3}$$

where

$x(t)$ = vector of composition perturbations throughout the column, dimension $x(t) = N$
$L(t)$ = reflux flow perturbation
$x_c(t)$ = perturbation of the composition on the control tray from the desired composition
A, B = process matrices with dimension $N \times N$ and $N \times 1$
$L_{\text{Max}}, L_{\text{Min}}$ = limits on the allowable reflux flow perturbations.

The solution of (1) subject to (2) and (3) is

$$L(t) = \begin{cases} L_{\text{Max}} & \text{if } \sum_{n=1}^{N} \beta_n x_n < 0 \\ L_{\text{Min}} & \text{if } \sum_{n=1}^{N} \beta_n x_n > 0 \end{cases} \tag{4}$$

where $\beta_n = \beta_n\,(A, B)$ = the controller parameter associated with the composition on tray n.

The control law given by (4) is nearly optimal in the sense that we would actually prefer to choose $L(t)$ so as to minimize $\int_0^\infty x_c^2(t)\,dt$ and instead we have only mini-

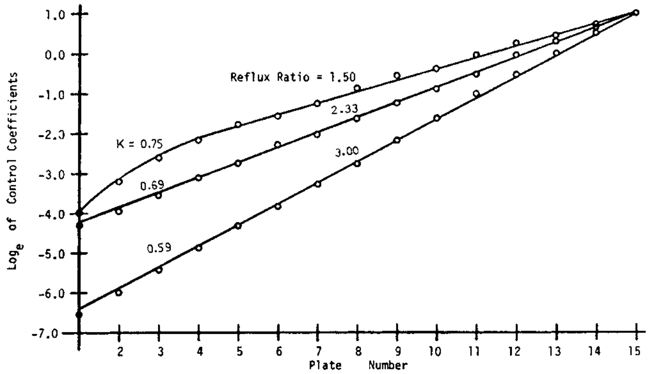

Fig. 1. Control coefficients for the pilot plant distillation column.

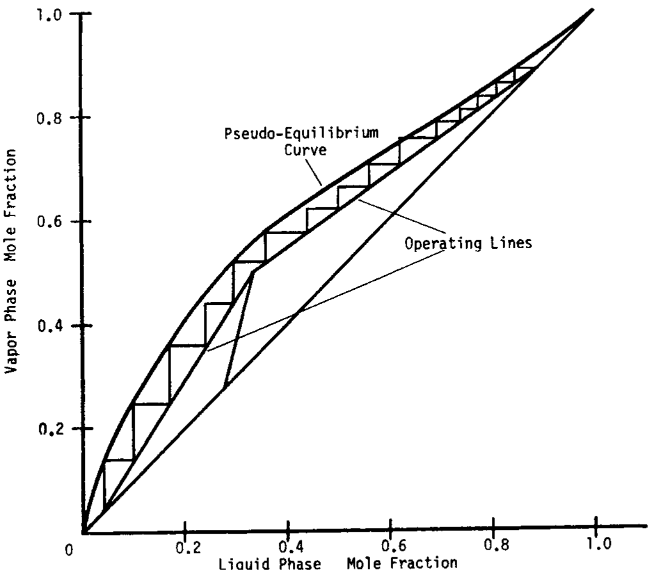

Fig. 2. McCabe Thiele diagram for the pilot plant distillation column.

mized its integrand because this leads to a much simpler control law (*1, 3*).

Figure 1 shows how the control coefficients β_n depend on stage position within the column. The coefficients were computed from the model given by Equation (3) using the method described in (*1*). The curve for a reflux ratio of 2.33 corresponds to the operating conditions given in Figure 2. Stages are numbered from the partial reboiler (# 1) to the top of the column (# 15). As in (*1*) the control stage was taken as the second stage from the top of the column rather than the top stage because the subcooled reflux entering the top stage makes temperature measurements there an unreliable indicator of concentration. The nearly linear nature of the curves in Figure 1 allow us to write

$$\beta_n = e^{K_1 n} \quad \text{or} \quad \beta_{n-1} = K\,\beta_n; \quad n = 2, \ldots N \qquad (5)$$

where $K \equiv e^{-K_1}$

The value of β_N can be chosen arbitrarily, as it does not affect the sign of Equation (4). Thus the control law depends only on the parameter K. Notice that as the reflux ratio increases, K decreases. This means that at high reflux ratios the on-off controller relies more heavily on measurements near the control tray.

In a binary system we can convert the composition measurements required by (4) into measurements of the temperature and pressure at each stage. Considering only perturbations in composition and pressure, we have

$$T_n = s_n\,x_n + q_n\,p_n \qquad (6)$$

where

T_n = temperature perturbation

p_n = pressure perturbation

$s_n = \dfrac{\partial \overline{T}_n}{\partial \overline{x}_n}$, pressure held constant

$q_n = \dfrac{\partial \overline{T}_n}{\partial \overline{p}_n}$, composition held constant

$\overline{(\cdot)}$ = variable $(\cdot)$ evaluated at steady state conditions

Solving for x_n and substituting into Equation (4) yields

$$L(t) = \begin{cases} L_{\text{Max}} & \text{if } \displaystyle\sum_{n=1}^{N} \frac{\beta_n}{s_n} T_n - \sum_{n=1}^{N} \frac{\beta_n\,q_n}{s_n} p_n < 0 \\ L_{\text{Min}} & \text{if } \displaystyle\sum_{n=1}^{N} \frac{\beta_n}{s_n} T_n - \sum_{n=1}^{N} \frac{\beta_n\,q_n}{s_n} p_n > 0 \end{cases} \qquad (7)$$

Assuming the validity of Equation (5), only knowledge of s_n, q_n, and K are needed to implement Equation (7). The parameters s_n and q_n are usually available from steady state data, and the appropriate value of K can be obtained by adjusting this parameter on-line to obtain a dead beat response to a step change in the set point. The tuning procedure is described in the section on experimental results.

This study used an atmospheric column, and so pressure fluctuations during a run lasting a few hours could generally be neglected. Hence, p_n was taken as zero for all n. In general, however, temperature measurements will have to be compensated for pressure fluctuations. Due to the summation term in Equation (7), one can expect that (7) will be somewhat less sensitive to noisy pressure measurements than a controller using only single pressure measurement. However, it also seems that (7) should be no less, or more, sensitive to real pressure fluctuations than a standard single measurement controller. Figure 3 is a block diagram of the final control system.

EXPERIMENTAL EQUIPMENT

Figure 4 is a schematic diagram of the pilot plant. The column is constructed of three flanged sections containing a total of fourteen plates. The liquid feed enters at either plate 5 or 7 (the plates are numbered from reboiler which is considered plate 1) while steam is fed to the heating coils in the reboiler at the bottom of the column. Feed is pumped from the feed supply tank, through the rotameter (2)† and flows down the stripping section to the reboiler (3) where a fixed level of liquid is maintained. Bottoms level is sensed by a differential pressure transmitter (4) and fed back to the pro-

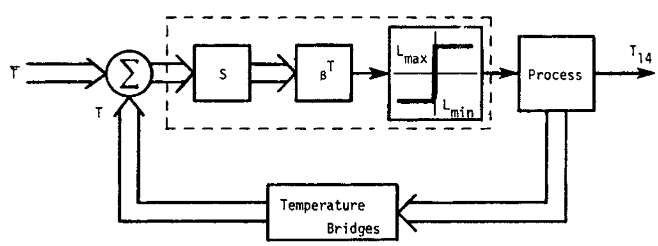

Fig. 3. On-off control system.

† The numbers in parentheses are used to identify the equipment in Figure 4.

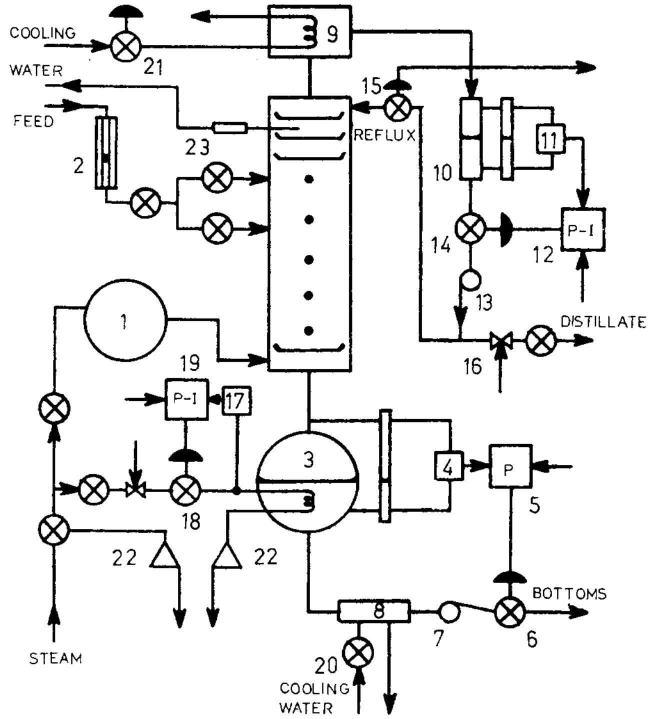

Fig. 4. Pilot plant schematic: 1. Kettle reboiler, 2. Feedline rotometer, 3. Reboiler, 4. Bottoms level DP cell, 5. Bottoms level controller (P), 6. Bottoms withdraw valve, 7. Bottoms pump, 8. Bottoms cooler, 9. Overhead condenser, 10. Distillate accumulator, 11. Accumulator level controller DP cell, 12. Accumulator level controller (P-I), 13. Distillate pump, 14. Distillate control valve, 15. Reflux control valve, 16. Back-pressure regulator, 17. Steam pressure transmitter, 18. Steam pressure control valve, 19. Steam pressure controller (P-I), 20, 21. Cooling water valves, 22. Steam traps, and 23. Thermistor bridge.

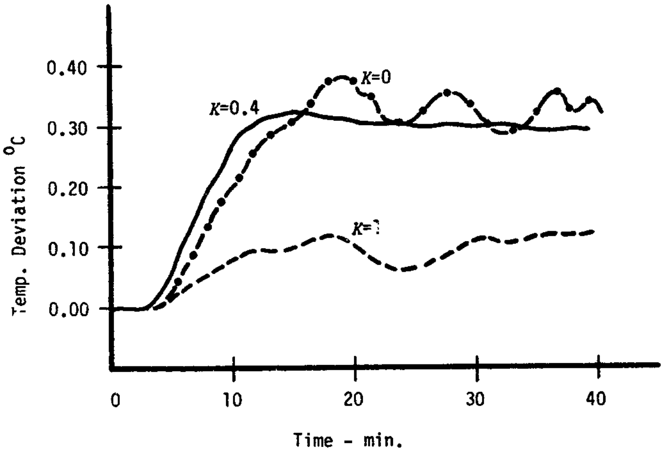

Fig. 5. On-off tuning curves.

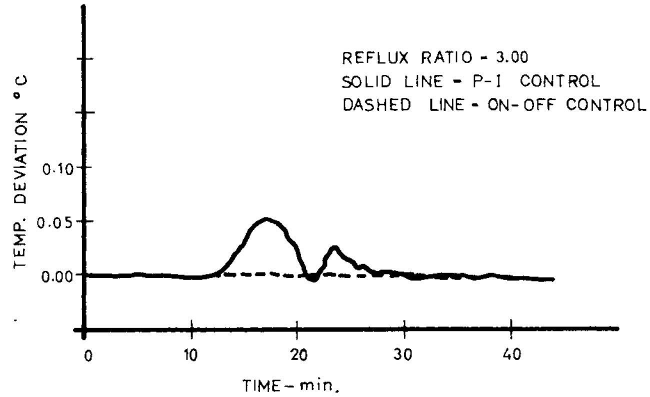

Fig. 6. Column response to feed flowrate disturbance.

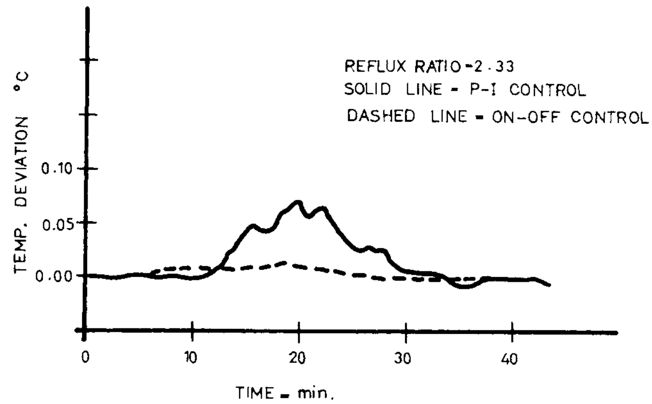

Fig. 7. Column response to feed flowrate disturbance.

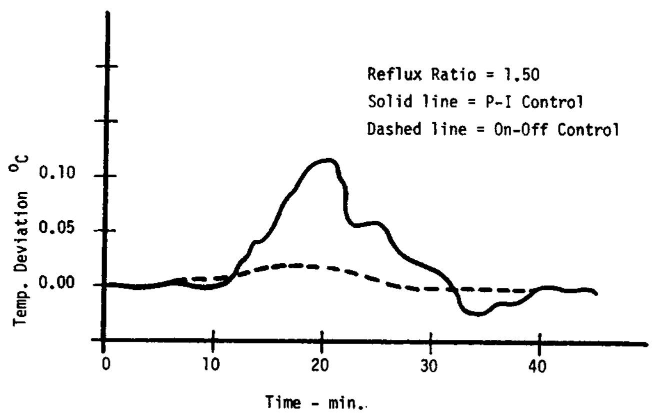

Fig. 8. Column response to feed flowrate disturbance.

portional controller (5) which adjusts the bottoms withdraw valve (6) so that the bottom product is withdrawn at a rate which keeps the reboiler level nearly constant. The bottoms stream is cooled (8) before being pumped into the receiving tank. The reboiler is steam heated and generates vapor which passes up the entire column. The vapors rising through the rectification section are completely condensed in the overhead condenser (9) and the condensate is collected in the accumulator (10). Accumulator level is maintained in the same manner as reboiler level with the exception that a proportional plus integral controller (12) is used to adjust the total condensate flow. The liquid withdrawn from the accumulator through the automatic valve (14) is then split into two streams; reflux and overhead product. The amount of the split is controlled by the reflux control valve (15). Steam pressure to the bottom of the column is maintained by a third analog control loop as shown in Figure 4.

The temperature on each plate and in the reboiler is sensed by Thermistors. The combined Thermistor and bridge circuits have a sensitivity to temperature variations of less than .01°C (2). Based on experiments with a constant temperature oil bath, it is estimated that the measurement noise contributed by the electrical system is on the order of ± 0.01°C.

The on-off control algorithm described in the preceding section and a standard proportional plus integral control algorithm were implemented using a GE 4060 process control computer. The real time control programs used in the study are described in detail in (2).

EXPERIMENTAL RESULTS

Before each run the column was manually brought up to the nominal steady state condition. The steady state temperature profile was then stored and control of the

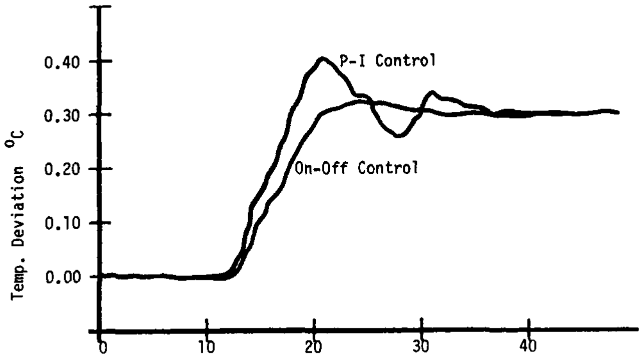

Fig. 9. Column response to change of state.

process transferred to the computer. The ordinate in Figures 5 through 9 is the response of the temperature on the control tray (2nd tray from the top) to various disturbances. A temperature change of 0.04°C corresponds to a change in composition of 0.01 mole fraction.

Figure 5 shows the effect of varying the tuning parameter K while the column is under on-off control. When K equals 1.0 equal weighting is assigned to each of the measurements. In this case the output is seen to drift slowly towards the set point. At the other extreme $K = 0$, zero weighting is assigned to every measurement except at the top plate and the control system is now the familiar single measurement, on-off controller. The output variable in this case exhibits limit cycle behavior. It was found that a rapid dead beat type of response is obtained with values of K between 0.45 and 0.6 when the column is operated at reflux ratios of between 1.5 and 3.0. This compares reasonably well with the range of K between 0.6 and 0.45 which was computed using the technique described in (*1*) (compare Figure 1).

The column response to a step disturbance in feed flow rate at three operating points is shown in Figures 6 through 8. The parameters of both the on-off and the P-I controller were tuned on line at the operating point corresponding to a reflux ratio of 3.0. For the P-I controller the gain K_p is 3.4% change in reflux valve position per °C change in temperature; the integral time constant; T_I is 6 min., and the interval between samples is 10 seconds. For the on-off controller K is 0.45. The on-off controller effectively suppressed the effects of the feed flowrate disturbance in all three cases, with no detectable steady state offset. P-I control was more sluggish in responding to the disturbance.

Figure 9 shows the transient response of the output variable as the column executes a controlled change of state. The dead-beat response is typical of the type of response encountered with on-off control. Here again the P-I controller is slower to respond and requires considerable overshoot in order to approach the settling time of the on-off controller. Efforts to eliminate the oscillatory behavior resulted in a highly overdamped system.

It is shown in (*2*) that for on-off control the number of primary measurements may be reduced without seriously degrading the system performance. Experiments were performed using zero weighting for plates in the stripping section. This is reasonable since the control coefficients for the feed plate and below are approximately three orders of magnitude smaller than the control plate coefficient. It was found that for corresponding operating points the reduced measurement case required the same value of the on-off tuning parameter as the full measurement case.

DERIVATION OF THE EXPONENTIAL RELATIONSHIP BETWEEN CONTROLLER PARAMETERS

In this section we show that the relationship between the control coefficients given by Equation (5) is generally valid for binary distillation columns and is not simply peculiar to the pilot plant used in this study. The approach taken is to obtain an analytical solution to the suboptimal control problem for an idealized process. We begin by replacing the traditional equilibrium stage model (which is a set of nonlinear differential difference equations) by the following linear partial differential equation model for a simple rectification section with a total condenser

$$H\frac{\partial x}{\partial t} = L\frac{\partial x}{\partial m} - V\frac{\partial y}{\partial m}; \quad 0 \leqq m \leqq M \tag{8}$$

$$y = k\left(x - \frac{\partial x}{\partial m}\right) + b \tag{9}$$

$$\begin{aligned} y(0,t) &= y_0(t) \qquad \text{a known function} \\ y(M,t) &= x(M,t) \\ x(m,0) &= f(m) \end{aligned} \tag{10}$$

Finite differencing the spacial derivatives of (8) and (9) on a unit mesh yields (11) to (13)

$$H\frac{dx_m}{dt} = L(x_{m+1} - x_m) - V(y_{m+1} - y_m) \qquad m = 0, \ldots, M-1 \tag{11}$$

$$y_m = k\,x_{m-1} + b \qquad m = 1, \ldots, M \tag{12}$$

$$y_M = x_M; \quad y_0 = y_0(t); \quad x_m(0) = f_m \qquad m = 1, \ldots, M \tag{13}$$

The above equations are the classical equilibrium stage representation of a rectification column with constant holdup, linear equilibrium relations, and constant molar overflow. Holt (*5*) has shown that the solutions obtained from Equations (8) to (10) are good approximations to the solutions of Equations (11) to (13) for columns with more than five stages.

Substituting (9) into (8) and then expanding for small perturbations in the liquid flow rate about the steady state operating point yields

$$H\frac{\partial x}{\partial t} = (\overline{L} - k\overline{V})\frac{\partial x}{\partial m} + k\overline{V}\frac{\partial^2 x}{\partial m^2} + \gamma(m)L \qquad 0 \leqq m \leqq M \tag{14}$$

$$\left.\begin{aligned} \left(\frac{dx}{dm} - x\right)_{m=0} &= 0 \\ \left(k\frac{dx}{dm} - (k-1)x\right)_{m=M} &= 0 \\ x(m,0) &= 0 \end{aligned}\right\} \tag{15}$$

where

$$\gamma(m) \equiv \frac{d\overline{x}}{dm} = ce^{-2\rho m}$$

$$c = \overline{x}(0) - \frac{(\overline{y}_0 - b)}{k}$$

$$\rho = \frac{\overline{L} - k\overline{V}}{2kV}$$

The nearly optimal control law which almost minimizes the functional $\int_0^\infty x^2(M,t)\,dt$ is obtained by using the extension of Bass' technique to distributed parameter systems as described in (4). It is shown in detail in (2) that the control which minimizes $x^2(M,t)$ at each instant of time is given by

$$L(t) = \begin{cases} L_{\max} & \text{if } \int_0^M \beta(m)x(m)dm < 0 \\ L_{\min} & \text{if } \int_0^M \beta(m)x(m)dm > 0 \end{cases}$$

where

$$\beta(m) = \int_0^M \gamma(n)p(n,m)dn \qquad (16)$$

The function $p(n,m)$ is given by the solution of

$$\mathcal{L}_n^* p(n,m) + \mathcal{L}_m^* p(n,m) = -\delta(M-n)\delta(M-m) \qquad (17)$$

where $\mathcal{L}_n^*$ = adjoint of $\mathcal{L}_n$, the spatial operator of (14) and (15)

$\mathcal{L}_m^* = \mathcal{L}_n^*$ with m replacing n

For our problem

$$\mathcal{L}_n^*(\cdot) = \left\{ -(\overline{L} - k\overline{V})\frac{\partial}{\partial n} + k\overline{V}\frac{\partial^2}{\partial n^2} \right\}(\cdot)$$

$$\left\{ \frac{\partial}{\partial n} + (2\rho - 1) \right\}_{n=0} (\cdot) = 0$$

$$\left\{ \frac{\partial}{\partial n} + (2\rho - k + 1) \right\}_{n=M} (\cdot) = 0$$

Solving (17) for $p(n,m)$, substituting into (16), and simplifying yields

$$\beta(m) \cong \sum_i D_i e^{(\rho+\omega_i)m/M} \qquad (18)$$

where $\omega_i = (\lambda_i^2 + 2\rho^2)^{1/2}$
λ_i = i^{th} eigenvalue of the operator $\mathcal{L}_n^* + \mathcal{L}_m^*$ of (19).

The parameters D_i in (18) depend on the process parameters (for example, λ_i, ρ, M, etc.) in a complicated manner (2), but numerical experiments show that they decrease very rapidly as the index i increases. For example, consider the process described by the McCabe Thiele diagram of Figure 10. The change in the magnitude of the parameters as the index i increases is shown in Table 1 and Figure 11. Also shown in Figure 11 are the values for the control coefficients β_m computed from the stage model given by Equations (11) to (13) using the method described in (1). The two computations agree amazingly well. It appears therefore that Equation (18) can be replaced by the *tuning relationship* given by Equation (5) for most cases of interest.

TABLE 1. PARAMETERS FOR EQUATION (18)

Index i	λ_i	D_i*	Process Parameters
1	1.94	1.0	$\rho = .6$
2	4.44	2.1×10^{-2}	$k = .75$
3	7.24	6.2×10^{-4}	$M = 7$

* The D_i have been normalized so that $D_1 = 1.0$.

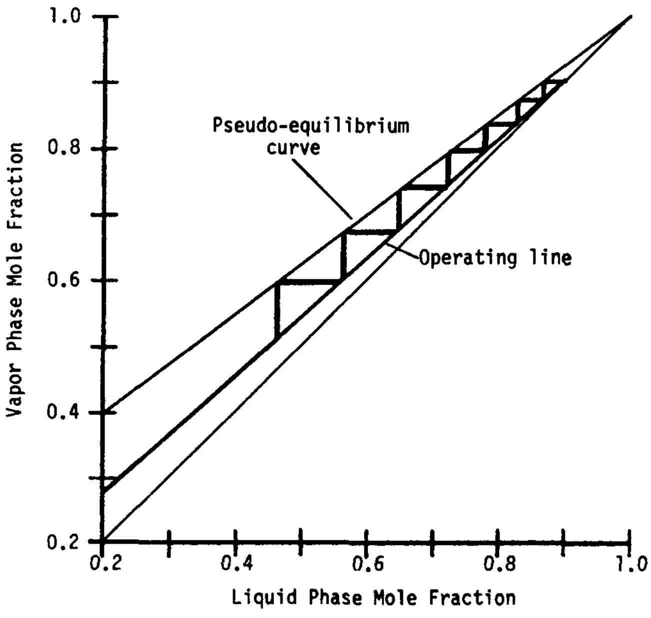

Fig. 10. McCabe Thiele diagram for example rectification section.

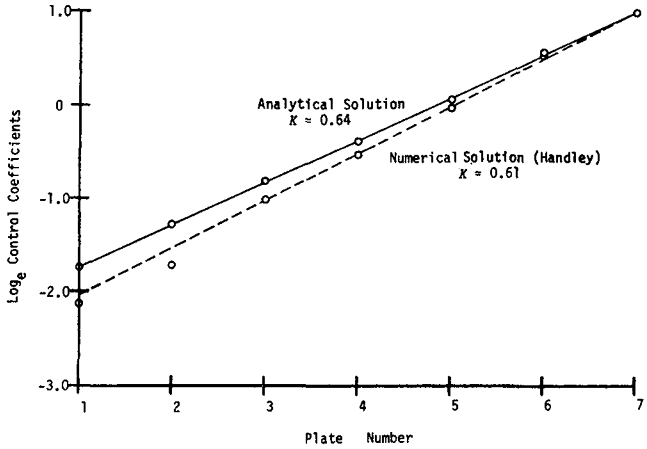

Fig. 11. Weighting function for example system.

CONCLUSIONS

The nearly optimal on-off control system described in (1) has been developed to the point where it can be applied to control the overhead composition of a binary distillation column with little more effort than that required to implement a simple proportional controller. In addition, the tuned on-off controller has been found to be significantly superior to a tuned proportional plus integral controller and to a single measurement on-off controller in the control of a pilot scale distillation column subject to feed flow disturbances and set point changes.

NOTATION

A, B = process matrices
β_n = nth control coefficient
$\gamma(m)$ = $\dfrac{d\,\overline{x}\,(m)}{dm}$, slope of the steady state composition profile

$\delta(n)$ = Dirac delta function $\int_{-\infty}^{\infty} \delta(n)\,dn = 1$

H = holdup per stage

k = slope of pseudo equilibrium curve

K_1, K = tuning parameter

L = liquid flow rate

$\mathcal{L}$ = linear operator

$\mathcal{L}^*$ = adjoint of $\mathcal{L}$

λ_i = ith eigenvalue of $\mathcal{L}_n^* + \mathcal{L}_m^*$

M = total number stages

ρ = a parameter $= \dfrac{\overline{L} - k\overline{V}}{2k\overline{V}}$

S = matrix of scaling factors, $s_{ii} = 1/s_i : S_{ij} = 0$

T_n = temperature of liquid stream x_n

x_n = liquid composition on the nth plate

y_n = vapor composition passing liquid stream x_n

V = vapor flow rate

A bar over a quantity indicates that it takes on only steady state values.

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Gas-Liquid Slug Flow with Drag-Reducing Polymer Solutions

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The two-phase slug flow regime in horizontal cocurrent gas-liquid flow was studied for the case of the liquid phase containing small amounts of polyacrylamide, a drag-reducing long chain polymer. The experimental work was performed in a 2.54 cm I.D. horizontal test section 10.7 m long.

Results indicated that two-phase drag reduction was greater than in single phase flow at the same superficial liquid velocities. By the use of pressure drop results with and without polymer additive, and a knowledge of slug geometry, frequency, and velocity, it was possible to separate approximately the contribution to the pressure drop of liquid wall friction and slug inertial effects. In all cases, the acceleration term was important and is the major energy term at most flow conditions. The technique may be of general usefulness in determining accelerational effects in two phase flows.

When drag reducing agents have been added to a horizontal slug flow, the pressure loss can be correlated successfully both by the Lockhart-Martinelli type of relationship or by a universal drag reduction curve of the type proposed by Virk.

Concurrent gas-liquid flow is frequently encountered in engineering processes. It occurs in boiler tubes, distillation columns, in polymer processing, and in chemical reactor applications. This type of flow has many unique features, and these must be evaluated in each situation. However, one phenomenon which is nearly always undesirable is the high axial pressure gradient, with a resultant substantial energy consumption per unit volume of liquid throughput.

It was observed by Toms (*1*) in 1948 that a substantial reduction of the frictional pressure gradient in one-phase turbulent flow could be achieved by the addition of dilute long chain polymers in solution, an effect now

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